

Guide to Writing MSc Dissertations

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Contents

1	What is expected	2
1.1	The investigation	2
1.2	The dissertation	3
2	How to go about your dissertation project	4
2.1	Planning	4
2.2	Studying and researching	4
2.3	Communicating with your supervisor	5
3	Structure of the dissertation	6
3.1	Format of the dissertation	6
3.2	The introduction	7
4	The bibliography and citing references	8
4.1	Essential citations	8
4.2	Helpful citations	9
4.3	Know what you cite	9
4.4	Bibliographic details	9
4.5	Citation styles	10
5	The writing process	11
5.1	Start	11
5.2	Revise	12
6	Writing mathematics	13
6.1	How to write mathematics	13
6.2	Useful L ^A T _E X hints	15
6.3	Citing with L ^A T _E X	16
	References	17

The dissertation project is an important part of postgraduate education. Most students are surprised how much they learn in the process, both in understanding and organising the material and in writing the dissertation.

This is a guide on how to write an MSc dissertation. It is written for Master's students at the Department of Mathematics at the London School of Economics. It is not an official document, but tries to provide help, and addresses common difficulties of students and concerns of their supervisors. Its aim is to lead you to good habits that are useful whenever you write and communicate.

1 What is expected

The dissertation is a significant component of the MSc degree. It has two parts: investigating and understanding a topic, and producing a coherent piece of text that describes the results of the investigation. Both parts are typically new for students, and highly instructive. In studying the topic, the student must work independently, understand texts that may be difficult or terse, and possibly solve unfamiliar problems that are not pre-chewed like classroom or textbook exercises. Writing up is even harder. The results of the investigation have to be explained clearly and informatively, in an appropriate tone and style, and in proper form. Writing well is hard work, and an activity that must be learned. The MSc dissertation provides such a learning opportunity.

1.1 The investigation

The dissertation project typically relates to the research interests of your supervisor. Common types of projects are:

- (a) a survey of a specific topic;
- (b) understanding and explaining a published, or about-to-be-published, research result;
- (c) writing a computer program and explaining the results of running it.

Sometimes, the project offers the student the opportunity for original research, for example proving a mathematical conjecture related to a result studied as in (b). Normally, an MSc project does not require you to produce original research, because the outcome is unknown and may be difficult to obtain. On the other hand, this is what research is about, so a good student may find this an interesting challenge.

The dissertation topic is always set so that a student can get a Distinction even without any original research contribution, provided the dissertation is well written; the MSc Handbook explains the criteria for marking a dissertation.

1.2 The dissertation

Solving the problems of the MSc project is only the first half of the work. The results of the investigation are reported in the MSc dissertation. In writing a dissertation you no longer just reproduce, you produce. It is the product that counts, not the effort.

The writing of the dissertation will be judged against a number of criteria, and your dissertation should meet the following standards. These standards concern:

- (a) *Proper form.* The dissertation has to follow a standard format for scientific communications. It has to have a title, introduction, sections describing the results, and a bibliography. It should be written in correct English and follow conventions of citation and of mathematical writing. Use English (UK) spelling, even if most of your sources use American spelling.
- (b) *Accuracy.* What is written must be logically coherent and correct. It must be clear which contributions are your own and which are taken from other sources. Negative results can and should also be reported; these can be parts of the investigation that are inconclusive, for example a conjecture that remains open.
- (c) *Readability.* The text should be a readable narrative, hopefully interesting, and above all clear. Assume a consistent level of what the reader already knows. You can assume that the reader has a general mathematical background, but you cannot assume that the reader is familiar with the papers that you write about. Also, it is not enough that an expert in the field can read “between the lines” that you have understood the topic. A non-expert has to be able to make sense of what you write as well. Making yourself clear is an important skill that you learn when writing the dissertation.

An even more basic requirement than the accuracy stated in (b) is that you *must not present someone else’s work as your own*. You may find a text somewhere that perfectly describes part of your subject. If you use that source without citing it, you commit *plagiarism* and violate basic ethics of scholarship, because you mislead your reader about your own contribution. Plagiarism is a severe offence, and the School has a detailed procedure and penalties for dealing with plagiarism.

If you are running short of time, and are concerned that your dissertation is not good enough, you should complete it as well as you can: cite the sources you use, and explain those results in your own words. Even in the (rare) event that your dissertation is graded as a Fail, you may still pass the degree as a whole.

Section 4 below describes how to cite sources correctly, including internet material, for which there is not yet a scholarly standard like for printed work. In general, say honestly what you do, for example “the exposition in this chapter follows [X, Section Y]”, which is also informative for your reader. You can then relax.

2 How to go about your dissertation project

This section gives suggestions on how you should pursue your MSc dissertation project to make it a success.

2.1 Planning

First, try to select a dissertation topic, and your supervisor, according to your interest in the general area, and according to what is likely to be expected of you. The particular topic that is offered may be unfamiliar to you, but your interest should grow once you start working on it. If it is a computer programming project, find out what you should do, in particular *with* the program once it is ready, because normally you have to write the dissertation about the use of the program, not about its code.

Be realistic about your own abilities. If you are expected to study difficult research publications, you should have some knowledge about the field (for example by having taken the supervisor's course), as stated in the prerequisites for the project. It is much better to study an easier research result, and understand and explain it well, than to understand a difficult paper poorly.

Start early, and reserve time for the project. The MSc project is a worth a quarter of your MSc degree, as much as two half-unit courses, and takes at least as much time. You should get started on the dissertation in Lent Term. Consult the "Instructions and Guidelines for the Dissertation in Applicable Mathematics" for the official timetable of the dissertation project.

You will need a major part of the Easter break, between Lent and Summer term, for exam preparation. Reserve the whole summer after the exams for the dissertation project. Do not go off to a job, or a vacation, unless you have essentially completed the work beforehand.

2.2 Studying and researching

Your supervisor is likely to provide at least one research paper as an introduction to the topic of your dissertation. Your first step is to read and understand this paper. Research papers are more terse than textbooks. When you read the paper, *understand the simple things first*. Create your own examples. You will also have to identify certain standard results that may be taken for granted in the paper, for example "Farkas' Lemma" in a paper on Linear Programming, even though this is not explicitly stated, and with which you may not be familiar. If you suspect that there is a standard result that you should know, look it up, with Google, wikipedia or in a textbook or handbook. Your supervisor will also be able to tell you. You can then proceed to the more difficult parts.

In the course of studying, you may have to decide that you have to limit your dissertation topic further in order to write on it successfully.

You should also get a feeling for the context of the research that you investigate, by looking at related work.

Finding and selecting references is an essential part of the scholarship that goes into your dissertation. Searching and understanding related papers is often part of the project, in particular for a dissertation that is a study of the literature as in 1.1(a).

Related papers are found, first of all, by following the references in the papers that you currently study, and their references. Related papers have often overlapping reference lists, which give you a feeling for the field and its important papers. If you explore the literature seriously, you will soon encounter papers that turn out to be irrelevant for your topic. You can safely delimit your subject if, in this way, you have looked at more papers than you cite.

In addition to following citations from one paper to another, you may look at other works by the same author. For recent work, that author may have a web page. Older papers can be found in the database of *Mathematical Reviews* [8] that gives summaries and possibly evaluations of papers; these are also worth checking for the papers that you already know. Citations that go forwards in time can also be found with *Mathematical Reviews* or with a *citation index* such as [6]. You type in an important paper that is central to your topic and find later references to that paper, which may be possibly interesting. Also, use Google or Google Scholar with various combinations of keywords about your topic.

One purpose of the search may be to establish that there is *not* much more to your subject than you have found already.

2.3 Communicating with your supervisor

- (a) You have to submit an *Initial Report* and an *Interim Report*. The purpose of these reports is to ensure that you understand the problem you are working on and do not get stuck on technical problems, in particular mathematical typesetting with L^AT_EX, in August when nobody is around to help you.
- (b) Research papers that you study are likely to be much more difficult than textbooks. Identify what you do and do not understand. Do not get stuck at a single sentence or paragraph of the paper that you study. Read on, with partial understanding on what the paper is about, then come back to the stumbling blocks. You may still not understand them. Meet your supervisor and explain: “here it says X but this does not make sense to me because...”. Your supervisor may not have a direct answer, but may be able to find it out with you.
- (c) Early on, try to write one or two *sample chapters*. These will test your writing abilities. One sample chapter can be a draft introduction that describes the problem and how you plan to address it. You will need such a piece of text anyhow as a starting point for an improved introduction. Moreover, defining the problem gives you a chance to clarify possible misunderstandings about the topic. A second sample chapter should describe an early part of your own study. Select a small part of the project, and understand and describe it. This also gets you started, both on working independently and on writing it up, and on talking to your supervisor.

As the project progresses, your supervisor may be willing to look at a *sample draft* of your dissertation. *Appreciate your supervisor’s time as a very limited resource.* Your super-

visor is not obliged to read full dissertation drafts in the first place, but does this to help you. Pay attention to the following:

- (d) Your draft should be free of obvious errors, in particular in punctuation, grammar, or citations. It is not only a lame excuse to say “oh, I haven’t checked it for typos yet”, but you make a bad first impression, and you distract your supervisor’s attention from the contents to trivial formal aspects. Get into the habit of writing English texts (including your emails) without errors. In general, make the best possible effort and do not leave it to others to fix your text.
- (e) Listen carefully to comments on your draft. You may get your draft back with a general comment, for example to be “more factual”. Clarify if this means that the general tone of the dissertation should change. If you get specific comments, follow them, but note that they are often only examples of what needs to be improved. Understand the spirit of these comments and change your text at other places where that kind of problem occurs.
- (f) Plan ahead. Find out when your supervisor is away. If you give your supervisor a draft of your dissertation a week before it is due for submission, you will at best get suggestions that you can implement in a few days. If you invest enough time early on in your MSc project, you should not encounter severe problems at this late stage.

3 Structure of the dissertation

This section gives guidelines on the form of the dissertation, namely on its general format, and on the particularly important introduction. The bibliography is also important and considered in the next section.

3.1 Format of the dissertation

The main sections of your text should be of roughly equal size. Sections that are reasonably balanced are easier to read. You may use subsections if they help locate a topic and make the table of contents more informative, but in general, use as few levels of subdivision as possible. In most cases, sections and, if necessary, subsections, suffice. If you start using sub-sub-sections, they tend to appear only in certain places so that the table of contents looks skewed, and because they are often short they make the page look cluttered.

As an example, Section 4 in this guide that deals with the bibliography is not made a subsection 3.3, in order to avoid sub-sub-sections.

Use only one type of paragraph. This paragraph does it wrongly. It has sentences that start on a new line because the writer is undecided whether to use a new paragraph or not.

Do not start a new line inside a paragraph because of a slightly new thought! A period ending a sentence has that purpose.

If your text contains figures and tables, they should have a caption (a short text explaining what they are about), and of course they must be referred to in the text.

Footnotes are added to the text as side remarks that can in principle be skipped. In general, footnotes are problematic because if they are meant to be read, then the reader is distracted by jumping to the bottom of the page and back to the main text. In that case, do not put the side remark into a footnote but include it in the text. Organise the thoughts in your sentences such that the distraction by the side remark is minimal, for example by putting it at the end of a paragraph.

Remarks in parentheses (like these four words) tend to distract and should be used sparingly; they can often be omitted, or the parentheses removed.

3.2 The introduction

The introduction is the most important section of your dissertation. Whenever you write a longer technical document, most readers never go beyond the introduction. If the introduction is good, they may feel encouraged to read on. The introduction is also the hardest part to write, and has to be rewritten several times.

It is therefore important that you learn how to write a good introduction. Most of the following hints apply to writing in general.

The introduction describes the area in which you are working, gives the basic definition and terminology, and sets out the fundamental results. If your dissertation contains a proof of a result, which may be yours or someone else's, then you should give the statement of the result in the introduction and explain its significance.

As a good rule for structuring any argument, in particular the introduction, it is useful to answer the sequence of questions *what – why – how*. Always state *what* you are talking about first before justifying it or diving into details.

The “what” part of the introduction summarises the contents of your dissertation. Ideally, you should be as informative as possible. Obviously you cannot say everything at once, so you may have to simplify. You may choose to tell a “white lie”, but you should try not to make statements that are wrong; for instance, you may by add a qualifier like “under certain reasonable assumptions”.

The introduction should always cite and, if possible, summarise relevant work done by others. This puts the work of the dissertation in context and allows the reader to judge the dissertation's contribution. If you can do so briefly, you may give a history of your subject first in order to explain what the current work is about. In that way, you simultaneously take care of the “what” and the “why” part.

Usually, the “what” part comes first, the “why” at a suitable time later. The “how” part should summarise the methods used in the dissertation, and possibly give further details.

If you present original research, it is good to explain the main ideas in the introduction, and make them sound as un-mysterious as possible. If this is done well in the introduction, the reader will be curious to read more about them. You should make it clear that you are the first person to have found something (if that is correct), but be careful and modest about it.

At any rate, make it clear in the introduction what your own contributions are, which may be original research or in terms of exposition. Do not be shy to state contributions that are small, for example “in Section 5 we illustrate Theorem X of [Y] with an example”.

The final paragraph of the introduction is typically a brief list of the sections of the dissertation and their contents.

The following is a list of common *mistakes* in an introduction and how to avoid them.

- (a) Exaggerated claims, for example “differential games are one of the most important tools of economics”. This may be your impression after studying differential games, but it sounds naive. Adopt a neutral tone, and remain careful and factual. The subject of the dissertation does not have to be declared as very important.
- (b) Assuming too much knowledge from your reader. You have immersed yourself in the topic for several months, but your reader has not. Be aware of that, and explain and introduce your topic in a comprehensible way.
- (c) An introduction that is an unclear medley of exposition, history of the subject, and a repetition of what others have done. A good way out of this is to deal with these aspects separately, in particular, to postpone the exposition to a main section. State early what *you* do in the dissertation. Suppose that the dissertation is mostly on a topic covered in paper X. You may choose similar opening sentences as paper X. However, when paper X says “We solve this problem as follows”, do *not* say “we”, but say instead “This problem is solved in [X] as follows . . .” and then state how you will explain the results of paper X in a later section of your dissertation.

In the writing process, the introduction can normally be finished only when the main text is complete because only then do you know its contents and structure. For your dissertation, try nevertheless to produce a draft introduction early on. You will get practice in writing, and gain valuable feedback on your view of the topic from your supervisor.

4 The bibliography and citing references

Citing references is part of any scholarly writing. In your dissertation, you have to demonstrate that you can cite properly. This section of the guide explains what and how you should cite.

4.1 Essential citations

It is *mandatory* that you cite your sources; otherwise, you plagiarise. You must not present something that you obtained from someone else as new. You also have to avoid making that impression inadvertently. If you use someone else’s wording verbatim, it is useful to say so explicitly, as in “The following definition is taken verbatim from [X].” If you base an entire section of your dissertation on some other work, you can explain this once at the beginning of your section.

The citation is part of the story that you tell. It is not enough to merely include the citation in the bibliography, because then it is not clear where you have used it. *Any cited work must be referred to in the main text.*

On the other hand, you only have to cite those works that are relevant for your work, not everything that you have read.

4.2 Helpful citations

Some citations help your reader to understand what you are doing. For example, if you state a new theorem that is similar to a theorem in a paper (which you may have already cited earlier) but which is different, say so explicitly. Otherwise, your reader may think that you have overlooked the similarity, or may not appreciate the difference.

Another type of citation is of material that you assume is known, or that you do not want to spend too much time on, where the reader can obtain further details. Here, it is good to cite standard reference books, rather than, for example, lecture notes from your home university, because this shows that you know and can judge your field. You can assume certain mathematical basics (in particular of linear algebra and calculus) without citing them. If you are in doubt, ask your supervisor.

4.3 Know what you cite

Do not use *second-hand* citations from other papers without obtaining and checking the cited works yourself; you can often get them online and do not even have to print them. First of all, the reference details (for example volume or page numbers) may be wrong and then you copy a mistake. In rare cases where you cannot get easily hold of a classic reference, for example [2], at least double-check the bibliographic details. Second, there are references with the same title, for example a technical report like [3] and later publication [4] which nevertheless differ, which you can only find out by getting hold of the reference itself. Third, even a brief look at the cited work tells you if it is relevant for your work. It may also be more informative than the secondary source where you got it from. An original research paper can be surprisingly readable because the author had to get something new across and tried to explain it well; it is worth having a look.

If you use a paper that does not have bibliographic details, for example a printout that your supervisor gave you for your dissertation preparation, you should find out how it can be obtained, and add information that may be missing, like a date. An internet search with the title or a sentence from the abstract will show if it is available online.

4.4 Bibliographic details

The publications that you cite in your dissertation are listed at the end in the bibliography, often entitled “references”. They should be listed *alphabetically* by author so that an entry is found quickly.

The bibliography is one of the first things a reader looks at. It places your work in context. An expert reader will recognise familiar references quickly, and also notice omissions. A non-expert reader may appreciate the bibliography as a way to learn more about the topic. A good bibliography indicates that you are in command of your subject.

For all reasons named so far, the bibliography provides you with an excellent opportunity to make a good first impression. You should therefore care about the formal details, and about what you cite.

Getting the formal details right sounds boring, but you have to do that only once. All bibliographic entries should be correct, complete, and consistent. The bibliography of this guide gives examples of journal articles [1][5], books [2], articles in edited volumes [7], conference proceedings [4], technical reports [3], and material from the internet [9][10]. The latter is often not permanent and should have an access date.

In general, a citation should be as complete and as informative as possible. If in doubt, a very complete bibliographic entry is provided by *Mathematical Reviews* [8]. For the article [1], this gives:

Audet, Charles; Hansen, Pierre; Jaumard, Brigitte; Savard, Gilles. Enumeration of all extreme equilibria of bimatrix games. *SIAM J. Sci. Comput.* 23 (2001), no. 1, 323–338. This is changed in this guide by replacing authors' first names with initials, and omitting the journal issue number ("no. 1") because journal pages are numbered per volume, not per issue.

After collecting the bibliographic details, you only have to work on making them look consistent. In the bibliography of this guide, book and journal titles (as found in a library catalogue) are capitalised and in italics, but article titles are not, and journal titles are not abbreviated.

4.5 Citation styles

You have to decide on how you cite your references in the text, and use that style consistently. There are essentially two possibilities, namely either by *author and year*, often called "Harvard style", or by *number* in square brackets.

An example of citing by author and publication year is Fortnow and Kimmel (1994), corresponding to the numbered citation [3] that this guide uses. Citing author and year in the text is informative, because a reader familiar with that reference does not even have to look at the bibliography. Moreover, you may often want to mention the authors anyway. When you cite by author and year, you do not have to number the references in your bibliography. See Section 6.3 for a bibliography in this style.

The only problem with citing author and year is that this becomes long when a paper is cited many times. This may be minimised in a single sentence or paragraph by arranging the citations suitably. If your paper talks about a reference many times, an acceptable way out of this may be to say "Fortnow and Kimmel (1994), henceforth abbreviated as [FK]", because presumably only very few citations will recur that often.

The “et al.” abbreviation is used for a paper with more than three authors, so [1] is cited as Audet et al. (2001). Moreover, you can also use it when you cite a three-author paper a second time.

With author and year citations, put the year parentheses even when the citation itself is in parentheses, as when you say (see Halmos (1970)). Reference to specific page numbers can be given as in Halmos (1970, p. 125). An example of multiple citations by the same authors is Fortnow and Kimmel (1994; 1998).

Alternatively, you can use *numbered* citations as in this guide, which are shorter. The references in the bibliography should always be sorted alphabetically, *not* in their order of citation in the main text, to keep the reference list informative. Use the `\cite` command in L^AT_EX to create these numbers, as explained in Section 6.3.

Citing a reference per number is done in square brackets as in [1]. More specific references can then be given as in [1, p. 333]. For that reason, I prefer to cite multiple papers as [1][5] and not [1, 5], although you may find the latter look nicer. As usual, you should follow a consistent style.

Information found on the internet is often transient. Give the full URL (web address) as well as the date when you accessed it. For example, the citation [10] was accessed online at http://en.wikipedia.org/wiki/List_of_Fumblerules on Oct 24, 2007, but this link is no longer valid, so the date is informative. You only have to give the URL for online material that is not easily accessed otherwise, so you do not have to give a URL for standard journal and conference publications (here, the URL of [5] is only given to help you find this article more easily).

5 The writing process

Your dissertation should read well and be clear and precise. Writing well requires effort and practice, which you acquire when writing your dissertation.

5.1 Start

The main writing effort goes into organisation and re-writing. It is impossible to write a perfect dissertation in one go. You have to start somehow, and will produce a version that you do not like. However, you need that mediocre version as a basis for writing a better dissertation.

You should enjoy writing and re-writing rather than being annoyed about it. Imagine that what you throw away counts as much towards your output as what you keep. In fact, it does, because it improves the quality of your text.

A good way to start is “brainstorming”. Collect your material, noting keywords for your ideas. Later on, keep a sheet of paper for additional ideas to put into the dissertation at some point.

Then sort your material. Writing can be viewed as organising material, down to the organisation of the words in a sentence. You have to bring a tangle of ideas into a linear sequence.

Each sequence of thoughts should follow a logical connection, with not too many jumps. Try to make a point, for example explaining a concept, only once in the main text. This will force you to think about the best place to make that point. Important things you will have to say twice, the first time in the introduction where you have to be brief, and a second time in the main text.

Always ask yourself: *What do I want to say?* In most cases, when you get stuck when writing, you do not know exactly what you want to say. Make that idea clear in your mind first, then write it. Often, you can be unhappy with a piece of text because one thought led to another, but you wound up somewhere unintended. If you know what you want to say, then you know early if you are going in the right direction.

Dedicate one paragraph to one topic at a time. Get directly to the point. Do not worry too much about gluing the new paragraph to the previous one. A clear structure holds the text together, not phrases. The resulting paragraphs will become “modular” and can typically be moved around easily.

Get into “writing mode” where you are agile and write and re-write quickly. Do not agonise over first sentences, which you will come back to later anyhow. Sometimes, you can get into “writing mode” by revising your recent text.

In the first version, it is best to just say what you want to say, possibly awkwardly, and without too much worry about repetition and elegance. Use the same noun in the next sentence, rather than a pronoun, because you can then move sentences around. Then move on. If you change a sentence too often, you no longer see how it reads normally. After some time has passed, you often find a better way of saying things, for example by breaking an awkward sentence in two.

5.2 Revise

After you have produced a first version, you will not like some things about it, and may be happy with other parts, for example with their logical order.

“I don’t like it” is a useful impression that you should analyse rather than persuading yourself of the opposite. Have you adopted the wrong tone (for example, talking down to your reader)? Are there conflicting goals in a certain piece of text (for example, two different types of audience)? Are you getting lost in details on a side point? Do you need a new piece of terminology here and should you introduce it much earlier? Most of all, does what you say *make sense*? Frequent “cut and paste” by computer can produce incoherent writing; it is often better to re-write.

You may have to scrap some parts of your draft and start over. Remember that the final product counts, which tends to be good only if you throw things away.

Other parts of your text may be at the right place and may be roughly right. Revise them as well. Shorten sentences by micro-organising them so that, for example, a concept is introduced at the end of a sentence where the next sentence can pick it up.

Cut fluff such as “it can therefore be observed that”. Shorten long and pompous phrases; compare “initial impressions can have an extraordinarily powerful impact” with “first impressions matter” (they do, by the way).

Polish sentences so that they can be read without stumbling. Avoid false leads. Often, a word or phrase can suggest another meaning that is not intended, forcing the reader to re-read. An example is the first sentence of this paragraph, where “Polish” can be misread as in “Polish sentences are longer than English ones”. Moreover, “without stumbling” is a “dangling participle” because it has no subject (it is certainly not the sentences that are stumbling).

Use correct English. Check the definition of a word with a dictionary. This is essential when you have found a word with a thesaurus. Do not mix singular and plural, or past and present. The passive sense should be avoided. And never start a sentence with a conjunction, separate main sentences by periods not commas. For further “fumble rules” see [10].

It is more important that your text is clear and precise, and to the point, than that it “reads smoothly”. You may say something at a place where it does not belong, or where it distracts. Check what happens if you omit it altogether. In that case, remove it, even if you have fallen in love with the expression that you found; perhaps you can use it at a more appropriate place.

Your final version should *scan* well. The reader should feel informed after a first look. The first sentences of each section, and in the same way the first words of a paragraph, should give an idea of what is coming.

Be *explicit* about the logic of your argument, so that the reader can follow it even without checking the precise details. Imagine your supervisor reading your dissertation at midnight on the underground train. In contrast, your text should not read as if it was written at midnight on the underground. If it reads effortlessly, your readers will notice its quality and appreciate the effort you have put into it.

6 Writing mathematics

We comment on special aspects of writing mathematics. The article [5] gives more details on how to write a long mathematical text, like a book. We also give some hints on the \LaTeX typesetting system, which are useful after you have started using \LaTeX .

6.1 How to write mathematics

The problem of writing mathematics is not much different from any communication, where you have to explain an idea. Consequently, you have to be as clear as possible, which in mathematics requires you to be *precise* as well.

In your dissertation, you communicate mathematics by English text, not by a mere sequence of equations and formulas. You have to explain what you are doing. A good principle of writing is to *speak to someone*, like your fellow students, or a mathematician who is not an expert in the field that you cover.

Use the editorial “we” rather than “I” which tends to sound arrogant. The best tone is *neutral*, as in “every vector space has a basis”, which can help you minimise the use of “we”. A comment such as “let us now define” is verbose and can often be replaced by the imperative “define”.

You typically give your text structure by breaking it into definitions, theorems, lemmas, examples, remarks, and so on, which are usually numbered so that you can refer to them later. Between these formal statements, explain in words what you are doing. A theorem or lemma summarises a result that can be used without having to refer to its proof. The proof hides technical details, and its beginning and end should be easy to find.

In your sentences, separate mathematical expressions by text, not just punctuation marks that are easily mistaken as part of a mathematical expression. A harmless example: Do not write “for all k, l , k even, $1 \leq k, l \leq n$ ”, but instead “for all k, l , where k is even and $1 \leq k, l \leq n$ ”. Avoid starting a sentence with a variable or other mathematical expression, like “ f is continuous”; instead, say “The function f is continuous”.

The mathematician Carl Friedrich Gauss said “we need notions, not notations”. Find good words for the concepts you introduce. Use those words and not some cryptic symbols that are easily forgotten.

Think hard about your notation. Do not name everything at once. Here is a typical bad example: “Consider a graph G with vertices v_1, \dots, v_n .” This instantly degrades the letter “ v ” to a coat-hanger for the subscript. Moreover, only the subscript carries any information, because you now have to talk about “vertices v_i and v_j ” rather than, say, u and v . Better say “Consider a graph G with n vertices”, if you need to refer to the number of vertices at all. Instead of naming the objects, here the vertices, it may be better to name the collection of objects, for example the “vertex set V ”. Then you can write an expression like $\sum_{v \in V} d(v)$, which is simpler than $\sum_{i=1}^n d(v_i)$. The alphabet is quite large if you do not waste it, so you can often use letters like x, y, z instead of sub- and superscripted, squiggled and otherwise decorated identical letters. This gives you a chance of keeping your formulas simple and readable.

Clarity comes before elegance. Allow for repetition: If two facts are essentially the same, it is better to repeat their definition than to paraphrase, which may confuse the reader. Say: “Let X be the set of strategies of player 1, and let Y be the set of strategies of player 2”. This is clearer than “Let X be the set of strategies of player 1. Player 2 chooses her actions from Y .” It is also not much longer than “Let $X(Y)$ be the set of strategies of player 1 (2)”, which has the danger that the reader thinks that there is a mathematical object $X(Y)$.

When you include *computer programs* in your dissertation (often in an appendix), make sure they are readable. Use a fixed-width font such as Courier. Explain your computer programs, in particular on a high level that says *what* they do, rather than how they do things.

6.2 Useful L^AT_EX hints

L^AT_EX is a typesetting system for mathematical text. A reasonably comprehensive guide is [9]. We give some hints which come up frequently, or which are useful but not so well known. Creating a bibliography and citations in L^AT_EX is explained in Section 6.3.

- All mathematical text should be typed between $\$. \cdot \$$ signs, including single letters. The variable x is typed as $\$x\$$. Instead of “the i -th component” say “the i th component”.
- Use `\ldots` for dots as in $\{1, \dots, n\}$, typed as $\$\{1, \ldots, n\}\$$.
- After a period, L^AT_EX produces a large white space because periods normally end sentences. This large space looks ugly after an abbreviation e.g. like here, which should be typed e.g. `\` to create a normal space. After capital letters, a normal space is generated anyhow, because they are normally initials, as in D. E. Knuth.
- The non-breakable space `~` helps avoid awkward line breaks as in, for example, “equation (9)”, which you can avoid by typing `equation~(9)`, `function~f`, and so on.
- The `\mid` command gives you a vertical bar surrounded by spaces, to be used in definitions of sets, as in $S = \{x \in X \mid f(x) \geq 0\}$, typed as $\$S = \{x \in X \mid f(x) \geq 0\}\$$.

Some useful packages to go into the preamble of your document (the preamble contains LaTeX commands typed between `\documentclass...` and `\begin{document}`):

- `\usepackage{mathptmx}` creates a document in Times font, with mathematics in *Times Italic*. This font is used in this guide.
- `\usepackage{amsmath}` gives you the `\mathbb` font for the real numbers \mathbb{R} , integers \mathbb{Z} , and so on. Then $f : \mathbb{R} \rightarrow X$ is typed as $\$f : \mathbb{R} \to X\$$.
- `\clubpenalty=10000 \widowpenalty=10000` prevents that a single first or last line of a paragraph appears alone at the bottom or top of a page, respectively.
- You can define your own LaTeX commands in the preamble of your document. An example is `\newcommand{\reals}{\mathbb{R}}`. Then type `\reals` instead of `\mathbb{R}`.
- `\usepackage[notcite,notref]{showkeys}` prints out labels introduced by `\label` to refer symbolically to equation and theorem numbers later on with `\ref`. This is useful for drafts; switch it off for your final version.
- Number definitions, theorems, lemmas, and so on, consecutively (possibly adding the section number), rather than by separate counters that make these statements hard to find. In the preamble, this is achieved by using a single theorem counter by typing
`\newtheorem{theorem}{Theorem}[section]`
`\newtheorem{lemma}[theorem]{Lemma}`
`\newtheorem{definition}[theorem]{Definition}`
- `\usepackage{hyperref}` must be used with `pdflatex` (which produces directly a PDF rather than DVI file) and creates links in the `\tableofcontents`, and for the `\cite`, `\ref`, and `\url` commands, as in this guide.

6.3 Citing with L^AT_EX

This section gives suggestions for creating a bibliography when you use the L^AT_EX system for typesetting mathematical text.

The following is an example of a bibliography for citing with author and year where references are not numbered.

Audet, C., P. Hansen, B. Jaumard and G. Savard (2001), Enumeration of all extreme equilibria of bimatrix games. *SIAM Journal on Scientific Computing* 23, 323–338.

Cournot, A. A. (1838), *Recherches sur les Principes Mathématiques de la Théorie des Richesses*. Hachette, Paris.

Fortnow, L., and P. Kimmel (1994), Beating a finite automaton in the big match. University of Chicago, Department of Computer Science, Technical Report TR-94-21.

Halmos, P. R. (1970), How to write mathematics. *L'Enseignement Mathématique* 16, 123–152.

Kuhn, H. W. (1953), Extensive games and the problem of information. In: *Contributions to the Theory of Games, II*, eds. H. W. Kuhn and A. W. Tucker, Annals of Mathematics Studies 28, Princeton Univ. Press, Princeton, 193–216.

The only problem is to make the first line of each reference stick out to the left. The following L^AT_EX code is one way to generate the above bibliography. Explanations are given afterwards.

```
{\small
\frenchspacing
\parindent=-2em\advance\leftskip by-\parindent

\strut \vskip-\baselineskip

Audet, C., P. Hansen, B. Jaumard and G. Savard (2001),
Enumeration of all extreme equilibria of bimatrix games.
\emph{SIAM Journal on Scientific Computing} 23, 323--338.

...

Kuhn, H. W. (1953),
Extensive games and the problem of information.
In: \emph{Contributions to the Theory of Games, II}, eds.
H.~W. Kuhn and A.~W. Tucker, Annals of Mathematics Studies 28,
Princeton Univ.\ Press, Princeton, 193--216.

}
```

If you care why this is done in this way, `\small` chooses a slightly smaller font size. With `\frenchspacing`, the blank space after a period is not larger than elsewhere, because abbreviations are common in references. With `\parindent=-2em...`, a “negative” indentation of the first line in the paragraph makes that line stick out to the left; the width 2em is 2 times the width of the “M” character in the current font. Finally, the change of the paragraph shape

does not work the very first time after a section heading like `\section*{References}`, so `\strut \vskip-\baselineskip` creates an invisible blank object called `\strut` and the `\vskip...` skips a blank line backwards so one does not see that unused paragraph. The whole thing is inclosed in braces `{ }` so that the change of font size and paragraph shape are kept local to that part of your file. Make sure that the closing brace `}` appears in a new paragraph, because otherwise the paragraph shape of the last reference (starting with “Kuhn...” above) is changed back prematurely.

You should type page ranges like 323–338 with a double hyphen 323--338, because the single hyphen is too short: if you type 323-338, you get 323-338.

In the citation style with numbered references, like in this guide, create a bibliography as follows. The argument 99 describes the width of the largest number that can occur, so replace that by 9 if you have less than 10 references.

```
\begin{thebibliography}{99}
\bibitem{Audet}
Audet, C., P. Hansen, B. Jaumard and G. Savard (2001),
Enumeration of all extreme equilibria of bimatrix games.
\emph{SIAM Journal on Scientific Computing} 23, 323--338.
...
\end{thebibliography}
```

Here, the `\bibitem{Audet}` gives a label that represents the automatically generated number for that reference. In the text, use the `\cite` command, as in `\cite{Audet}`, which produces “[1]”, or, with more details, `\cite[p. ~333]{Audet}` produces “[1, p. 333]”.

I recommend to type the bibliographic entries (the “`\bititems`”) manually as in the above example. There is a more elaborated `bibtex` package, which creates consistent bibliographic entries from a database. However, if you use this, you will still have to tweak the database. For example, article titles are converted to lower case, so to make sure that proper names are capitalised in the database entry, you have to type things like `{N}ash equilibrium`. Non-standard entries such as contributions to edited volumes like [7] are particularly tiresome to get right in `bibtex`.

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